The Charge Response of
a Meson-Correlated Relativistic Fermi Gas *

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Abstract

The quasielastic longitudinal electromagnetic response function R_L is studied within the context of a model that extends our previous investigations of pionic correlations and currents. Four mesons are now employed (π, ρ, σ and ω, via the Bonn potential) and the many-body dynamics are extended to the full random-phase approximation built upon a Hartree-Fock basis. Wherever possible the Lorentz covariance of the problem is respected. The first three energy-weighted moments of the reduced response are computed, namely, the zeroth moment (Coulomb sum rule), the first moment (related to the position of the quasielastic peak) and the second moment (related to the peak width or variance). We discuss how with a modest downward adjustment of the Fermi momentum it is possible to obtain the expected zeroth and second moments; this implies that the nuclear system has a lower density than that required when using the free relativistic Fermi gas and accordingly that the interaction effects are weaker than one might initially find.

1 Introduction

In past work [1, 2] we have addressed the problem of setting up both the *longitudinal* (R_L) and the *transverse* (R_T) responses that enter in the expression of the inclusive cross section for inelastic scattering of electrons from nuclei. The inelasticity domain of concern here is that of the quasielastic peak (QEP). Our studies were based on the relativistic Fermi gas (RFG) model and the guiding principles of our investigation were relativistic covariance and global gauge invariance (fulfillment of the continuity equation) — indeed much effort was expended in adhering to these two fundamental constraints. In particular, the choice of the RFG model was prompted not only by the assumption that the nuclear surface is likely of secondary importance in very inelastic nuclear phenomena, but also by the recognition that the two principles referred to here are easier to implement in a translationally invariant system than in a finite nucleus.

Moreover, in our previous work we resorted to the Hartree-Fock (HF) and anti-symmetrized random phase approximation (RPA) many-body frameworks, *adding* their contributions to get the nuclear responses. These procedures are correct when the treatment is limited to first-order contributions in a perturbative expansion, an approach that has some merit when the emphasis is being placed on achieving a consistent treatment of the forces and currents (the meson-exchange currents (MEC) are in fact usually expressed through first-order perturbative diagrams) and when the interaction between nucleons is carried solely by the *pion* whose associated correlations are weak enough to make a first-order treatment quite reliable. Indeed, comparison with the infinite-order calculations showed this to be the case, at least for momentum transfers higher than roughly 300 MeV/c [2].

In the present work we extend the previous investigations in two directions by

i) including, beyond the π, additional mesons (namely the ρ, σ and ω) in the description of the nuclear force, and

ii) calculating the nuclear electromagnetic responses in the fully antisymmetrized RPA scheme *evaluated in a HF basis*.

Clearly, item ii) is implied by i), since the forces carried by the ρ, σ and ω are too strong to be dealt with in first order. On the other hand, the requirement of global gauge invariance is now much harder to implement in a consistent manner and we leave this issue to be dealt with in a future paper.

Here we address the specific problem of finding how the forces between the nucleons shape the quasielastic response in the range of momenta between 300 MeV/c and 1 GeV/c, confining ourselves to the charge response R_L (the transverse one, R_T, is treated in a companion paper). To assess the impact of the forces on R_L we focus not only on the longitudinal response itself, but on three derived quantities as well, namely the sum rule, the mean energy and the variance of the response (we shall

later provide precise definitions for these quantities). Note that these observables relate only to the many-body content of R_L, which should therefore be carefully disentangled from nucleonic aspects of the problem as we shall see in detail.

In performing the present investigation an underlying theme has been the role played by the pion. Apart from the self-evident importance of the pion in the nuclear dynamics, our interest has also been prompted by the findings of ref. [3], where it has been shown that, owing to its isovector nature, the pion acting alone induces a remarkable enhancement of the longitudinal vector weak neutral current response. This in turn entails a huge effect on a certain observable related to the asymmetry as measured in parity-violating inclusive electron scattering. Our wish to explore whether or not pionic effects survive when other powerful components of the nucleon-nucleon interaction are switched on is therefore understandable.

Significantly, we have found that the role of the pion, far from being "suppressed", is actually "enhanced" by the shorter-range part of the nuclear force through an interference mechanism hidden in the complexities of the charge response of the correlated RFG. On the other hand, it is well-known that pionic effects in the ground-state energy of a spin-isospin saturated system like the RFG or a closed-shell nucleus are suppressed, because the key ingredient, namely the Hartree contribution, is eliminated by the spin-isospin traces.

It thus appears that in nuclear matter the ground state and the QEP involve different aspects of the dynamics, namely that the action of the pion in the ground state is hampered, while in the QEP it is enhanced. This is a long-standing question that does not appear to have received unambiguous experimental resolution; however, a recent reanalysis [4] of polarized $(\vec{p}, \vec{n})$ experiments at LAMPF shows that the data are compatible with a softening of the spin-longitudinal response induced by the pion, as predicted many years ago [5].

Since the developments and results presented in the body of this work are quite involved, in the following paragraphs we summarize some of the salient findings before entering into those more detailed discussions. In sects. 2 and 3 we build the HF mean field utilizing as input the π, ρ, σ and ω pieces of the well-known relativistic Bonn potential [6]. We show that the HF field is strong and induces a very substantial hardening of R_L. This arises mainly from the tadpole diagram via the cooperative action of the σ and the ω, with however an additional sizable contribution having the same sign and stemming from the ρ meson through the "oyster" (Fock) diagram. Worth noticing is that the momentum dependence of the tadpole diagram, and hence the hardening of the charge response, is of relativistic origin.

As we shall see, we are able to account quite accurately for the action of the HF field on the nuclear response simply by shifting the energy ω transferred to the nucleus by an amount $\Delta\omega$ and by changing the bare nucleon mass m_N into a dressed one, m_N^*. It turns out that $\Delta\omega$ and m_N^* depend upon the momentum transfer q and the Fermi momentum k_F. This result is the key for obtaining RPA

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results in a HF basis without having to resort to quite heavy computations. The strength of the mean field up to the largest momentum considered here, $q = 1$ GeV/c, is also reflected by the variance of the HF longitudinal response (essentially the half-maximum width) which is significantly larger than that occurring in the RFG model, but for the lowest momenta.

In sect. 4 we investigate the roles played by the ring and exchange correlations in R_L. We deal exactly with the rings, whereas the exchange diagrams are treated in the framework of the continuous fraction method truncated at the first iteration. This scheme actually provides exact results for zero-range forces and also turns out to be quite accurate for finite-range interactions as long as the momentum transfer is not too small (say $q \geq 300$ MeV/c)

In the first part of sect. 4 we consider only diagrams going forward in time (anti-symmetrized Tamm-Dancoff (TD) approximation) and, to assess the impact of the mean field on the TD results better, these are presented with fermions propagating both freely and in the presence of a HF field. In the TD scheme (and as well in RPA) a struggle among the different contributions to the charge response takes place. Specifically the battle arises from the interplay between the direct (ring) and exchange diagrams and among the various mesons carrying the force. To unlock this complex situation it might help to recall that

a) the momentum behaviour of the ring and exchange diagrams is rather similar, both of them decreasing with q and becoming much reduced for $q > 500$ MeV/c;

b) generally, the light (heavy) mesons are more effective in transmitting the interaction at small (large) q, providing of course that the coupling constants are large enough.

Thus, considering first the ring diagrams, it is found that they indeed contribute sizably up to quite large transferred momenta and are obviously supported only by the σ and ω mesons. These dramatically reshape the free charge response in the isoscalar channel when acting alone, leading to the occurrence of collective modes lying above (the ω) and below (the σ) the energy range of the QEP that carry a large fraction of the charge strength (hence leading to a huge depletion of R_L inside the QEP). However, when acting together, the σ and the ω interfere so strongly that the "ring" collective modes disappear and the only remnant of the σ–ω interplay is a still appreciable damping (but for the lowest energies) and flattening at 300 MeV/c (but not for larger momenta) of R_L. In addition a *softening* of R_L takes place — this is large at 300 MeV/c, still significant at 500 MeV/c and then fades away at higher momentum transfer in accord with a). On the other hand, in accord with b) the σ wins over the ω, as it also does for the ground-state binding energy.

Turning to the exchange, it is contributed to by all of the mesons. Furthermore it opposes the action of the rings, thus leading in accord with b) to a hardening of R_L,

which shows up as a dramatic peak in the response at $q = 300$ MeV/c, with some shadow of it remaining even at 500 MeV/c. This peak corresponds to an "exchange" collective mode embedded in the particle-hole (ph) continuum that arises from the repeated exchange of the attractive π, ρ and σ mesons, which are more powerful than the ω at these momenta inside a ph ring. Of considerable significance in this connection is the role of the pion and, to a much lesser extent, of the rho: indeed, *without the pion* the exchange peak would *not* appear at all!

Actually, when the ring and the exchange act together, the repulsive peak disappears, although some hardening of R_L remains at 300 and 500 MeV/c, thus showing the dominance of the exchange correlations over those from the ring diagrams. At larger momenta the ring and exchange diagrams, beyond being quite small, almost completely cancel each other.

Summarizing: the TD collective effects in the charge channel arise basically from the exchange diagrams. They show up essentially through a hardening of the charge response at intermediate momenta (due to the attractive σ, but particularly to the π), which persists up to about 500 MeV/c. However, as we shall see, the TD correlations appear to be somewhat overcome by the HF field: thus the TD longitudinal response evaluated in a HF basis, while being appreciably depleted at intermediate momenta, is actually shifted at higher frequencies for all of the transferred momenta considered by an amount which is rather constant but for the lowest momenta.

In the second part of sect. 4 the diagrams going backward in time are explored (antisymmetrized RPA). The ground-state correlations (gsc) thus introduced appear to strengthen the action of the attractive ring diagrams somewhat, while at the same time reducing the contribution of the repulsive exchange. Indeed the "exchange peak" at $q = 300$ MeV/c, referred to above in the TD scheme, now no longer shows up and is instead replaced by a "ring peak", associated with the σ meson and appearing in the direct channel. Yet the non-linearity of the RPA is such that, even in the presence of gsc, the exchange term still prevails over the direct one when they are combined together and what remains is actually an appreciable hardening. In addition to this, the RPA correlations at momenta up to about 500 MeV/c (and likewise the TD ones, but more markedly so) have a tendency to quench and flatten R_L, particularly when evaluated in a HF basis, namely they produce trends that appear to be supported by some experimental evidence [7]-[12]. At larger momenta, the contribution of the gsc essentially vanishes, as can also be inferred from the observation that here the sum rule is well fulfilled.

The important role that the pion also plays in the RPA framework should be emphasized. Indeed, without it the *ring* peak would survive the confrontation with the exchange contribution: accordingly, the RPA response in a free basis, and in a HF one as well, would still display a peak on the low-energy side of the response. This would be entirely out of touch with reality, if only the σ and the ω were to act, and still there in a scheme encompassing the ρ as well. Thus, the statement that

the RPA charge response requires the pion appears to be justified.

In summary, the RPA correlations are found to yield an R_L that is appreciably less hardened than the one obtained in the TD scheme, but somewhat more quenched and with a slightly larger variance. This finding is in accord with the general result that the gsc lower (enhance) the response when the latter is hardened (softened) in order to comply with the energy-weighted sum rule. Finally, as in TD, the HF mean field still tends to play a prominent role within the RPA and again the role of the pion is crucial.

From our analysis of the longitudinal response based on the HF–TD and HF–RPA schemes with the Bonn potential as an input, we thus conclude that the correlations' effects are very substantial and to a large extent appear to disrupt the RFG predictions. One is accordingly led to address the question whether or not part of the interaction might be accounted for by "renormalizing" the only parameter characterizing our many-body system, namely the Fermi momentum k_F. Our choice, namely $k_F = 225$ MeV/c, has been dictated by the analysis of the ^{12}C data where such a value turns out approximately to account over a range of momenta for the width of the longitudinal response when the latter is analyzed in terms of the RFG model. Since the width at half-maximum together with the area and position of the QEP are the central features of the inclusive charge response (or, in general, of *any* response function), we have searched within the scheme of our meson-correlated RFG for a k_F capable of accounting for the experimental half-width or, which is roughly the same thing, of yielding the same half-width as the RFG with $k_F = 225$ MeV/c. In other words we have applied what can be defined as a principle of the *invariance of the variance*, the latter being of course related to the half-width of the response. The integrated response and peak position are then computed using this renormalized model.

The results are discussed in sect. 5. We have been successful in adhering to this principle, providing we let k_F vary slightly with q. The values of k_F necessary for achieving this turn out always to be lower than 225 MeV/c by about 10% and to be gently increasing with q (*i. e.,* the larger the value of q, the smaller the required renormalization of the Fermi momentum). Because of the non-linear dependence on k_F of the interaction effects in our model, the impact of the correlations on R_L is overall found to be appreciably diminished. Note that, due to the different ranges of the forces carried by the different quanta, the heavier mesons are those most affected by the procedure outlined above, whereas the pion emerges almost unscathed from the renormalization of k_F.

Whether the results obtained in the *invariance of the variance* scheme lead to closer contact with data or not is a question that remains to be answered in detail. This we have not done in the present work partly because we have not yet completed the evaluation of the MEC contributions, which should of course be added before attempting in-depth comparisons with the data, but additionally because two particle-two hole $(2p - 2h)$ excitations cannot be ignored in the QEP domain and

should be included as well in the analysis.

2 The Hartree-Fock field

Two diagrams contribute to the HF field, namely the direct (sometime referred to as *tadpole*) and the exchange (sometime referred to as *oyster*) diagrams displayed in Fig. 6. We have calculated their contribution to the nucleon self-energy utilizing the Bonn potential, which is derived through a non-relativistic expansion of the diagrams corresponding to the exchanges of the π, ρ, σ and ω mesons (actually the σ meson should be viewed as a fictitious entity corresponding to the exchange of a correlated pair of S-wave pions). Here we have neglected the small η and δ contributions in ref. [6] and for clarity retained only the largest pieces of the potential. It reads

$$V = V^\pi + V^\rho + V^\sigma + V^\omega, \tag{1}$$

where

$$
\begin{aligned}
V^\pi &= \left(V_S^\pi\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_T^\pi\, S_{12}\right) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tag{2}\\
&\quad V_S^\pi(q) = V_T^\pi(q) = -\frac{g_\pi^2}{12 m_N^2} \Gamma_\pi^2(q) \frac{q^2}{q^2 + m_\pi^2}, \\
V^\rho &= \left(V_0^\rho + V_S^\rho\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_T^\rho\, S_{12}\right) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tag{3}\\
&\quad V_0^\rho(q, P) = \left[g_\rho^2 \left(1 + \frac{3}{2}\frac{P^2}{m_N^2}\right) - \frac{g_\rho(g_\rho + 4 f_\rho)}{8 m_N^2} q^2\right] \Gamma_\rho^2(q) \frac{1}{q^2 + m_\rho^2} \\
&\quad V_S^\rho(q) = -2 V_T^\rho(q) = -\frac{(g_\rho + f_\rho)^2}{6 m_N^2} \Gamma_\rho^2(q) \frac{q^2}{q^2 + m_\rho^2}, \\
V^\sigma &= V_0^\sigma \tag{4}\\
&\quad V_0^\sigma(q, P) = g_\sigma^2 \left[-1 + \frac{1}{2}\frac{P^2}{m_N^2} - \frac{q^2}{8 m_N^2}\right] \Gamma_\sigma^2(q) \frac{1}{q^2 + m_\sigma^2}, \\
V^\omega &= V_0^\omega + V_S^\omega\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_T^\omega\, S_{12} \tag{5}\\
&\quad V_0^\omega(q, P) = g_\omega^2 \left[1 + \frac{3}{2}\frac{P^2}{m_N^2} - \frac{q^2}{8 m_N^2}\right] \Gamma_\omega^2(q) \frac{1}{q^2 + m_\omega^2}, \\
&\quad V_S^\omega(q) = -2 V_T^\omega(q) = -\frac{g_\omega^2}{6 m_N^2} \Gamma_\omega^2(q) \frac{q^2}{q^2 + m_\omega^2}.
\end{aligned}
$$

In the above S_{12} is the usual tensor operator and

$$\Gamma_\alpha(q) = \frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + q^2} \tag{6}$$

the standard meson-nucleon form factor. Moreover, $q \equiv |\mathbf{q}|$ is the momentum exchanged between the two nucleons and $P \equiv |\mathbf{P}|$ is the average of their relative incoming and outgoing momenta (as illustrated in Fig. 6, $\mathbf{P} = (\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{k}_1' - \mathbf{k}_2')/4$).

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meson	T	J^π	m (MeV/c)	Λ (MeV)	$g^2/4\pi$	$f^2/4\pi$
π	1	0^-	138.03	1300	14.9	–
ρ	1	1^-	769	1300	0.95	$0.95 \times (6.1)^2$
σ	0	0^+	550	2000	7.7823	–
ω	0	1^-	782.6	1500	20	–

Table 1: The quantum numbers, the masses, the cut-off parameters and the coupling constants of the mesons included in the Bonn potential. Note the two coupling constants characterizing the ρ.

The **P**-dependent terms in the ρ, σ and ω potentials clearly correspond to a non-local interaction in coordinate space and their important impact on the dynamics of the system will be discussed later. We have omitted the spin-orbit terms ($\propto (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{q} \times \mathbf{P}$) because they do not contribute to the responses in an infinite system.

For the convenience of the reader we report in Table 1 the quantum numbers, the values of masses and cut-offs and the strengths of the couplings for all of the mesons entering into the potential.

Note that in our approach, as in refs. [1, 2], the short-range physics is again embedded in phenomenological vertex form factors which cut off the nucleon-nucleon interaction in a spatial region of size $\sim 1/\Lambda$ around each nucleon. Whether this is adequate or whether ladder diagram contributions to the responses should be explicitly accounted for in the QEP domain is unclear. This will need to be addressed in future work.

In connection with the HF field, we recall that only the σ and ω contribute to the tadpole term. Introducing dimensionless parameters

$$\eta_F = \frac{k_F}{m_N}, \quad \lambda_\alpha = \frac{\Lambda_\alpha}{m_N}, \quad \mu_\alpha = \frac{m_\alpha}{m_N}, \tag{7}$$

and measuring both the momentum and the self-energy in units of the nucleon mass ($\beta = k/m_N, \Sigma(k) = m_N \widetilde{\Sigma}(\beta)$), the corresponding real and energy-independent self-energies read

$$\widetilde{\Sigma}_\sigma^H(\beta) = \frac{8}{3}\frac{g_\sigma^2}{4\pi^2\mu_\sigma^2}\left(\frac{\lambda_\sigma^2 - \mu_\sigma^2}{\lambda_\sigma^2}\right)^2 \eta_F^3\left(-1 + \frac{3}{40}\eta_F^2 + \frac{1}{8}\beta^2\right) \tag{8}$$

$$\widetilde{\Sigma}_\omega^H(\beta) = \frac{8}{3}\frac{g_\omega^2}{4\pi^2\mu_\omega^2}\left(\frac{\lambda_\omega^2 - \mu_\omega^2}{\lambda_\omega^2}\right)^2 \eta_F^3\left(1 + \frac{9}{40}\eta_F^2 + \frac{3}{8}\beta^2\right). \tag{9}$$

On the other hand all of the mesons contribute to the exchange (*oyster*) term and

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the associated self-energies, which are again real and energy-independent, read

$$\widetilde{\Sigma}_\pi^F(\beta) = \frac{3}{8}\frac{g_\pi^2}{4\pi^2}\left[\mathcal{G}_\pi(\beta) + \mathcal{G}_\pi(-\beta)\right] \tag{10}$$

$$\widetilde{\Sigma}_\rho^F(\beta) = -\frac{3}{2}\frac{g_\rho^2}{4\pi^2}\left[\mathcal{F}_\rho(\beta) + \mathcal{F}_\rho(-\beta)\right]$$
$$+\frac{3}{16}\left(5\frac{g_\rho^2}{4\pi^2} + 12\frac{f_\rho g_\rho}{4\pi^2} + 4\frac{f_\rho^2}{4\pi^2}\right)\left[\mathcal{G}_\rho(\beta) + \mathcal{G}_\rho(-\beta)\right] \tag{11}$$

$$\widetilde{\Sigma}_\sigma^F(\beta) = \frac{1}{2}\frac{g_\sigma^2}{4\pi^2}\left[\mathcal{F}_\sigma(\beta) + \mathcal{F}_\sigma(-\beta)\right]$$
$$+\frac{5}{16}\frac{g_\sigma^2}{4\pi^2}\left[\mathcal{G}_\sigma(\beta) + \mathcal{G}_\sigma(-\beta)\right] \tag{12}$$

$$\widetilde{\Sigma}_\omega^F(\beta) = -\frac{1}{2}\frac{g_\omega^2}{4\pi^2}\left[\mathcal{F}_\omega(\beta) + \mathcal{F}_\omega(-\beta)\right]$$
$$+\frac{5}{16}\frac{g_\omega^2}{4\pi^2}\left[\mathcal{G}_\omega(\beta) + \mathcal{G}_\omega(-\beta)\right], \tag{13}$$

where

$$\mathcal{F}_\alpha(\beta) = \frac{\lambda_\alpha^2 + \mu_\alpha^2}{\lambda_\alpha}\mathrm{tg}^{-1}\left(\frac{\eta_F + \beta}{\lambda_\alpha}\right) - 2\mu_\alpha\mathrm{tg}^{-1}\left(\frac{\eta_F + \beta}{\mu_\alpha}\right)$$
$$+\frac{1}{2\beta}(\beta^2 - \mu_\alpha^2 - \eta_F^2)\ln\frac{(\eta_F + \beta)^2 + \lambda_\alpha^2}{(\eta_F + \beta)^2 + \mu_\alpha^2} \tag{14}$$

and

$$\mathcal{G}_\alpha(\beta) = \lambda_\alpha(\lambda_\alpha^2 - 3\mu_\alpha^2)\mathrm{tg}^{-1}\left(\frac{\eta_F + \beta}{\lambda_\alpha}\right) + 2\mu_\alpha^3\mathrm{tg}^{-1}\left(\frac{\eta_F + \beta}{\mu_\alpha}\right)$$
$$+\frac{1}{2\beta}(\eta_F^2\mu_\alpha^2 - \beta^2\mu_\alpha^2 + 2\lambda_\alpha^2\mu_\alpha^2 - \lambda_\alpha^4)\ln[(\eta_F + \beta)^2 + \lambda_\alpha^2]$$
$$+\frac{\mu_\alpha^2}{2\beta}(\beta^2 - \mu_\alpha^2 - \eta_F^2)\ln[(\eta_F + \beta)^2 + \mu_\alpha^2]. \tag{15}$$

We thus see from the above that the Hartree field stemming from the action of the σ and of the ω mesons displays a term that is quadratic in the momentum. The origin of this lies in the pieces of V_0^σ and V_0^ω that are proportional to $(q/2m_N)^2$ and $(P/2m_N)^2$, namely, to the parameters characterizing the non-relativistic expansion of the Bonn potential. In this connection note that the average between the initial and final relative momenta of the interacting nucleons in the Hartree diagram is

$$\mathbf{P} = \frac{1}{2}(\mathbf{k} - \mathbf{k}'), \tag{16}$$

$\mathbf{k}$ being the momentum of the propagating nucleon (particle or hole) and $\mathbf{k}'$ the momentum flowing in the bubble of Fig. 6. Accordingly, in principle $\mathbf{P}$ is unbound.

However, in practice, in order that the truncation of the non-relativistic expansion of the Bonn potential makes sense, both $|\mathbf{P}|$ and $|\mathbf{q}|$ should not exceed, roughly, the nucleon mass m_N.

Because of its quadratic dependence on the momentum the Hartree field can be accounted for by an effective mass (of course, but for the constant terms which enter into (8) and (9)):

$$m_N^* = \frac{m_N}{1 + \dfrac{\rho}{4m_N^3}\left[\dfrac{3g_\omega^2}{\mu_\omega^2}\left(\dfrac{\lambda_\omega^2 - \mu_\omega^2}{\lambda_\omega^2}\right)^2 + \dfrac{g_\sigma^2}{\mu_\sigma^2}\left(\dfrac{\lambda_\sigma^2 - \mu_\sigma^2}{\lambda_\sigma^2}\right)^2\right]}, \tag{17}$$

where

$$\rho = \frac{2k_F^3}{3\pi^2} = m_N^3\frac{2\eta_F^3}{3\pi^2} \tag{18}$$

is the uniform density of the Fermi system. The effective mass is displayed as a function of k_F in Fig. 6, where the individual contributions to m_N^* arising from the σ and the ω are shown as well. It is worth noting that the σ and the ω cooperate in dressing the bare mass of the nucleon whereas, as is well-known, they fight each other in accounting for the binding energy of nuclear matter. Clearly, this outcome reflects the *opposite* sign of the constant terms and the *same* sign of the quadratic terms of the self-energies in (8) and (9).

Turning to the Fock contribution, the nearly constant behaviour as a function of the momentum of the combined Fock self-energy of the σ and ω (see Fig. 6) is quite striking — note that a constant here produces no effect on the HF response of the system. On the other hand the Fock self-energy of the ρ is sizable: actually, if acting alone, being repulsive, it would harden the charge response with respect to the free case by about 20 MeV at $q = 500$ MeV/c for $k_F = 225$ MeV/c. It clearly cannot be incorporated in a constant effective mass over a wide range of momenta because, far from being quadratic, the Fock self-energy of the ρ displays a rather cumbersome momentum dependence (see eq. 11). Finally, we recall that in the *oyster* diagram $P = 0$.

In order to appreciate the dynamical impact of the HF field, let us now calculate the longitudinal electromagnetic response in the HF approximation. For this purpose we start by recalling its well-known expression [13], namely

$$\begin{aligned}R_L^{\mathrm{HF}}(q,\omega) &= \frac{3\pi^2 A}{k_F^3}f_L^2(Q^2)\int\frac{d\mathbf{k}}{(2\pi)^3}\theta(k_F - k)\theta(|\mathbf{k} + \mathbf{q}| - k_F)\\ &\quad\times\delta\left[\omega - \frac{q^2}{2m_N} - \frac{\mathbf{q}\cdot\mathbf{k}}{m_N} - \Sigma^{\mathrm{HF}}(|\mathbf{k} + \mathbf{q}|) + \Sigma^{\mathrm{HF}}(k)\right],\end{aligned} \tag{19}$$

A being the nuclear mass number (here and in the following we take $N = Z = A/2$), Q the four momentum $(\omega, \mathbf{q})$, $f_L(Q^2)$ the longitudinal γNN vertex and Σ^{HF} the HF

self-energy. It is interesting that (19) can be expressed as an integral over the effective mass according to [13]

$$R_L^{\text{HF}}(q,\omega) = \frac{\xi_A}{8\pi^2} \frac{1}{\eta_F^3 \kappa m_N^2} f_L^2(Q^2) \widetilde{R}_L^{\text{HF}}(q,\omega), \tag{20}$$

with $\kappa = q/2m_N$, $\xi_A = 3\pi^2 A$ and

$$\widetilde{R}_L^{\text{HF}}(q,\omega) = \left\{ \theta(\eta_F - 2\kappa) \int_{\eta_F - 2\kappa}^{\eta_F} d\beta\, \beta \right.$$
$$\left. + \theta(\eta_F - \kappa)\, \theta(2\kappa - \eta_F) \int_{2\kappa - \eta_F}^{\eta_F} d\beta\, \beta \right\} m_N^*(\sqrt{4\kappa^2 + \beta^2 + 4\kappa y'})$$
$$+ \left\{ \theta(\kappa - \eta_F) \int_0^{\eta_F} d\beta\, \beta + \theta(\eta_F - \kappa)\, \theta(2\kappa - \eta_F) \int_0^{2\kappa - \eta_F} d\beta\, \beta \right\} m_N^*(\sqrt{4\kappa^2 + \beta^2 + 4\kappa y''}). \tag{21}$$

In the above

$$m_N^*(\beta) = \frac{m_N}{\left| 1 + \dfrac{1}{\beta} \dfrac{d\widetilde{\Sigma}^{\text{HF}}(\beta)}{d\beta} \right|} \tag{22}$$

is the momentum dependent effective mass of the nucleon, whereas y' and y'' are roots of the equation

$$\eta_F \psi - y - \frac{1}{2\kappa}[\widetilde{\Sigma}^{\text{HF}}(\sqrt{4\kappa^2 + \beta^2 + 4\kappa y}) - \widetilde{\Sigma}^{\text{HF}}(\beta)] = 0, \tag{23}$$

satisfying the constraints

$$(\eta_F^2 - 4\kappa^2 - \beta^2)/4\kappa \le y' \le \beta \qquad \text{and} \qquad -\beta \le y'' \le \beta. \tag{24}$$

It is of importance to realize, in connection with the basic result (21), that in R_L^{HF} (and also in the transverse HF response) only the *particle* (and not the *hole*) effective mass explicitly appears.

Notice that the relativistic content of our approach is contained first of all in the relativistic scaling variable of the Fermi gas ψ. For the latter we use, rather that the exact one [14], the approximate but quite accurate relativistic expression

$$\psi = \frac{1}{\eta_F} \left[\frac{\lambda(\lambda + 1)}{\kappa} - \kappa \right], \tag{25}$$

with $\lambda = \omega/2m_N$. Next, relativity is also embedded in the longitudinal γNN vertex. For this we again employ the approximate but almost exact [1] expression

$$f_L^2(Q^2) = (1 + 2\lambda) \left\{ \frac{1}{1 + \tau}[G_E^{m_t}(\tau)]^2 + \tau[G_M^{m_t}(\tau)]^2 \frac{\eta_F^2}{2}(1 - \psi^2) \right\} \tag{26}$$

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q (MeV/c)	$\bar{A}\,(10^{-2})$	$\bar{B}$	$A\,(10^{-2})$	B	$m_N^*(p)/m_N$
300	-1.671	0.246	-0.961	0.146	0.77
500	-1.671	0.246	-0.424	0.116	0.81
800	-1.671	0.246	1.034	0.085	0.85
1000	-1.671	0.246	1.466	0.081	0.86

Table 2: The dimensionless parameters characterizing the biparabolic nucleon self-energy discussed in the text (see (27) and (28)). Note the variation of the particle self-energy with the momentum transfer q, reported in the last column. The effective mass of the particles *below* the Fermi surface (holes) turns out to be $m_N^*(h) = 0.67\,m_N$.

$G_{E,M}^{m_t}$ being the proton and neutron (with isospin projections m_t) electric and magnetic Sachs form factors and $\tau = |Q^2|/4m_N^2$.

Before displaying the results for R_L^{HF}, we now briefly outline an approach that greatly simplifies the RPA calculation of the nuclear responses based on a HF (rather than on a free) basis. This approach stems from the observation that the total HF nucleon self-energy induced by the π, ρ, σ and ω mesons, while not simply parabolic in the momentum, nevertheless lends itself to an interpolation with *two* parabolas, one below (hole states) and one above (particle states) the Fermi momentum. Accordingly, we set

$$\widetilde{\Sigma}^{\mathrm{HF}}(\beta) = \bar{A} + \bar{B}\beta^2, \qquad \text{for } \beta \leq \eta_F \tag{27}$$

and

$$\widetilde{\Sigma}^{\mathrm{HF}}(\beta) = A + B\beta^2, \qquad \text{for } \beta > \eta_F, \tag{28}$$

B and $\bar{B}$ being both positive. However, while $\bar{A}$ and $\bar{B}$ are truly constant, A and B are in fact momentum transfer dependent, since they are fixed by fitting the particle self-energy over the range of momenta required for the calculation of a given nuclear response. This range, of course, varies with q. As already noted, the procedure outlined here allows a quite accurate fit of the self-energy entering into the charge response.

In Table 2 we give the values of the parameters appearing in (27) and (28) for $q = 300$, 500, 800 and 1000 MeV/c.

Now with the self-energy (27-28) the solution of eq. (23) reads

$$y = \frac{\lambda}{\kappa}\frac{1}{1+2B} - \kappa + \frac{1}{1+2B}\frac{1}{\kappa}(a + b\beta^2), \tag{29}$$

with $a = (\bar{A} - A)/2$ and $b = (\bar{B} - B)/2$, and the constraint (24) (non-Pauli-blocked

region) is easily shown to require

$$\beta \geq \left| \frac{1 - \sqrt{1 - \frac{4b}{(1+2B)\kappa} \left[\frac{1}{(1+2B)\kappa} (\lambda + a) - \kappa \right]}}{\frac{2b}{(1+2B)\kappa}} \right|. \tag{30}$$

Then, by expanding the square root in leading order and noticing that (22) with the self-energy (28) yields the expression

$$m_N^* = \frac{m_N}{1 + 2B} \tag{31}$$

for the particle effective mass, we obtain for the lower limit of the momentum integration over the effective mass in (21) the value

$$\beta_0 = \frac{1}{1 + 2B} \left| (\lambda^* + a^*) \frac{1}{\kappa^*} - \kappa^* \right| \tag{32}$$

in the non-Pauli-blocked region. In (32) the star reminds us that the effective mass (31) should be used in the definition of the dimensionless energy and momentum transfer variables and $a^* = (1 + 2B)a$.

Finally, by inserting (32) in the expressions (20) and (21) for the longitudinal response, it is immediately apparent that one gets *for the HF response the same expression as for the free response but for the replacement of the bare nucleon mass m_N with the dressed one m_N^* and for the shift*

$$\lambda \to \lambda^* + a^* \tag{33}$$

of the energy.

Identical results can be shown to hold in the Pauli-blocked region, although in this case, owing to the energy shift (33), the response obtained with the approximation outlined here no longer exactly vanishes as λ goes to zero as it should. On the other hand we should not trust the physics of the RFG in the Pauli-blocked region anyway.

The above very useful and appealing results are crucially dependent upon whether the leading-order expansion of the square root in (30) is sufficient or not in providing a good representation for the lower limit of the momentum integral entering into the expression of R_L. That this is indeed the case will be shown in the next section.

3   The charge response in HF

We have calculated R_L in the HF scheme with the self-energies discussed in sect. 2 for $q = 300$, 500, 800 and 1000 MeV/c. The results are displayed in Fig. 6 where, in addition to the charge response of the free Fermi gas, three cases are shown as well, corresponding to the HF evaluation of R_L with

i) the true self-energy Σ^{HF};

ii) the self-energy fitted with a double parabola, as explained in the previous section;

iii) the analytic method, which we recall amounts to *dressing the nucleon mass and shifting the energy.*

Two features are immediately apparent from the figure. The first relates to the outstanding agreement between the results for R_L obtained with the three prescriptions referred to above. The expansion of the square root in (30) indeed appears to be warranted over a large span of momentum transfers. The second feature relates to the systematic hardening of the response induced by the HF mean field when compared with the free response. This appears to be substantial, is persistent as q grows and, as already pointed out, stems mostly from the quadratic piece of the Hartree self-energy of the σ and the ω.

In this connection, it is worth recalling that for local static potentials the Hartree self-energy *does not* contribute to the response. It does so in our case because the potentials mediated by the σ and the ω also depend on the average between the initial and final relative momenta of the two interacting nucleons, P, as seen from the non-relativistic reduction of the Bonn potential at order $O(1/m_N)$.

To gain further insight into the impact of the HF field on the nuclear responses and to pave the way for a similar analysis of the additional correlations we shall consider later, we now investigate the sum rule Ξ, the mean energy $\bar{\lambda}$ and the variance $\bar{\sigma}$ of R_L. These quantities should relate to the many-body content *only* of the nuclear responses, which should accordingly be disentangled beforehand from the complete charge response. To achieve this we follow the pattern established in ref. [3], where it has been shown within the context of the RFG that a *reduced charge response* $S^L(\kappa, \lambda)$ can be introduced, having an energy integral (the *Coulomb sum rule*) equal to one for $\kappa \geq \eta_F$. It is defined as follows:

$$S^L(\kappa, \lambda) = \frac{v_L R_L}{X'_L}, \tag{34}$$

where

$$v_L = \left(\frac{Q^2}{q^2}\right)^2 \tag{35}$$

is the usual leptonic factor and

$$X'_L \equiv \frac{\mathcal{N} X_L}{(\kappa \eta_F^3 / 2\xi_F)(\partial \psi / \partial \lambda)} \; . \tag{36}$$

In the above, $\xi_F = \sqrt{1 + \eta_F^2} - 1$,

$$\frac{\partial \psi}{\partial \lambda} = \frac{\kappa}{\tau} \frac{\sqrt{1 + \xi_F \psi^2 / 2}}{\sqrt{2\xi_F}} \left[\frac{1 + 2\lambda + \xi_F \psi^2}{1 + \lambda + \xi_F \psi^2}\right] \tag{37}$$

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$$= \frac{\kappa}{\eta_F \tau} \left(\frac{1 + 2\lambda}{1 + \lambda} \right) + \mathcal{O}[\eta_F^2] \tag{38}$$

and

$$X_L = v_L \frac{\kappa^2}{\tau} (G_E^2 + W_2 \Delta) \tag{39}$$

with [14]

$$W_2 = \frac{1}{1 + \tau} [G_E^2 + \tau G_M^2], \tag{40}$$

$$\Delta = \frac{\tau}{\kappa^2} \left[\frac{1}{3} (\varepsilon_F^2 + \varepsilon_F \Gamma + \Gamma^2) + \lambda(\varepsilon_F + \Gamma) + \lambda^2 \right] - (1 + \tau) \tag{41}$$

and

$$\Gamma \equiv \max \left\{ (\epsilon_F - 2\lambda), \ \kappa \sqrt{1 + \frac{1}{\tau}} - \lambda \right\}. \tag{42}$$

As in (36) $\mathcal{N}$ reminds us that in taking the ratio (34) a term with $\mathcal{N} = Z$ should be added to a term with $\mathcal{N} = N$ in both the numerator and denominator.

In terms of the reduced response (34), the Coulomb sum rule, the mean energy and the variance, namely related to the zeroth, first and second moments of the charge response, then read as follows:

$$\Xi^L = \int_0^\infty d\omega \, S^L(\kappa, \lambda), \tag{43}$$

$$\bar{\lambda}^L = \int_0^\infty d\omega \, \lambda \, S^L(\kappa, \lambda), \tag{44}$$

and

$$\bar{\sigma}^L = \sqrt{\int_0^\infty d\omega \, S^L(\kappa, \lambda)[\lambda - \bar{\lambda}^L]^2}. \tag{45}$$

In discussing our results, we recall that the HF sum rule should be identical to the RFG one. This is indeed seen to occur in Fig. 6a, where the momentum behaviour of Ξ_{HF}^L and Ξ_{RFG}^L is displayed. In Figs. 6b and 6c we display the momentum behaviour of $\bar{\lambda}_{\mathrm{HF}}, \bar{\lambda}_{\mathrm{RFG}}$ and $\bar{\sigma}_{\mathrm{HF}}, \bar{\sigma}_{\mathrm{RFG}}$, respectively, for the charge response.

It clearly appears from Fig. 6b that up to the largest momenta considered the HF field hardens the charge response as previously mentioned. With respect to $\bar{\sigma}$, it can be shown for $\kappa \geq \eta_F$ in the RFG framework that it is related to the width $\Delta\omega = m_N(4\kappa\eta_F/\sqrt{1 + 4\kappa^2})$ of the response region according to [15]

$$\bar{\sigma} = \frac{1}{2m_N\sqrt{5}} \frac{qk_F}{\sqrt{m_N^2 + q^2}} + O(k_F^2)$$

$$= \frac{1}{\sqrt{5}} \frac{\kappa\eta_F}{\sqrt{1 + 4\kappa^2}} + O(\eta_F^2) \tag{46}$$

$$= \frac{1}{4\sqrt{5}} \frac{\Delta\omega}{m_N} + O(\eta_F^2).$$

Remarkably, in Fig. 6c one observes that in spite of the dramatic hardening of the charge response induced by the HF field, which *grows* with q, the difference between the variance of R_L^{HF} and of R_L^{RFG} is quite small at low momenta ($\sim 5\%$), then it increases with momentum transfer up to about $q = 500$ MeV/c ($\sim 20\%$), to decrease again for larger momenta where the difference stabilizes around a value of 10%.

Thus, notwithstanding the violation of the energy-weighted and energy-squared weighted sum rules induced by the HF mean field, which is in accord with a general theorem on the contributions of the Feynman diagrams to the sum rules [16], yet it appears that these violations tend to compensate each other in arriving at $\bar{\sigma}_{\mathrm{HF}}$. It is as if the system reacts against those interactions that attempt to change the width of the charge response or, equivalently, the Fermi momentum k_F, which in turn fixes the density for a homogeneous system.

4   The RPA longitudinal response

In this section we explore the influence on the charge response of correlations between nucleons acting beyond the mean field. It might be useful in this connection to relate the ingredients used in the present work to Feshbach's scheme [17], which splits the Hilbert space of the nuclear Hamiltonian into a P-sector, where the *slow* nuclear motion occurs, and a Q-sector, where instead the *fast* motion associated with short-range correlations between nucleons takes place. In this framework the HF field and the RPA correlations are naturally included in the P-sector, whereas the $2p-2h$, $3p-3h$, *etc.,* types of excitations mainly belong to the Q-sector. Indeed the occurrence of the latter requires a large amount of energy which is mostly supplied by the violent nucleon-nucleon interaction at short range and short-range encounters between nucleons are presumably comparatively rare for kinematics in the QEP region. Therefore, our aim will be to calculate the polarization propagator $\Pi(Q)$ in RPA on a HF basis while ignoring the Q-sector contributions in the present work.

For this purpose, we start by recalling the link between $\Pi(Q)$, which for a homogeneous system only depends upon the two variables $|\mathbf{q}|$ and ω, and the *ph* Green's function G^{ph}, namely

$$\Pi(Q) = i \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} G^{\mathrm{ph}}(k+Q,k;p+Q,p) \tag{47}$$

(spin-isospin indices are here neglected for simplicity). Therefore, in order to get Π, one should first solve the Galitskii-Migdal equation [18] for G^{ph} which reads in the RPA framework

$$G^{\mathrm{ph}}(k+Q,k;p+Q,p) = -G(p+Q)\,G(p)\,(2\pi)^4\delta(k-p)$$

$$+iG(k+Q)\,G(k)\int \frac{d^4t}{(2\pi)^4}\left[U^{ph,\mathrm{dir}}(Q) - U^{ph,\mathrm{exch}}(k-t)\right]G^{\mathrm{ph}}(t+Q,t;p+Q,p),$$

$$\tag{48}$$

where the single-particle Green's functions G are meant to describe the fermion propagation in the HF field.

Clearly, the zeroth-order iteration of (48), when inserted into (47), just yields the polarization propagator for the non-interacting relativistic Fermi gas $\Pi^0(Q)$. The first iteration of (48), on the other hand, provides, via the direct ph matrix element of the interaction $U^{ph,\mathrm{dir}}$, the first ring (Fig. 6a) of the ring expansion and, via the exchange ph matrix element $U^{ph,\mathrm{exch}}$, the first (Fig. 6b) of the infinite series of exchange terms. When the two expansions are grouped together, the fully antisymmetrized Π^{RPA} on a HF basis is obtained. Note that the two first-order terms of the RPA series already embody contributions going both forward (TD) and backward (gsc) in time.

In this paper, rather then directly solving eq. (48), we resort to the equivalent continuous fraction representation of $\Pi^{\mathrm{RPA}}(\mathbf{q}, \omega)$ [19, 20, 17]. A detailed analysis of the method will be given in ref. [21], where, in particular, it will be shown how the ground-state correlations can be incorporated in the treatment (see also ref. [22]). Here, to help in understanding the present calculation, we merely recall the basic formulas.

In ref. [19] it had been shown that the polarization propagator in the TD approximation and in the first-order continuous fraction expansion is

$$\Pi^{\mathrm{TD}}(\mathbf{q}, \omega) \cong \frac{\Pi^0_R(\mathbf{q}, \omega)}{1 - V(\mathbf{q})\Pi^0_R(\mathbf{q}, \omega) - [\Pi^{(1)}_R(\mathbf{q}, \omega)/\Pi^0_R(\mathbf{q}, \omega)]}, \tag{49}$$

where the subscript R means that only the retarded component of the free propagator is included and where

$$\Pi^{(1)}_R(\mathbf{q}, \omega) = -\int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} G^0_R(k_1 + Q) G^0_R(k_1) G^0_R(k_2 + Q) G^0_R(k_2) V(\mathbf{k}_1 - \mathbf{k}_2). \tag{50}$$

Moreover $V(\mathbf{q})$ and $V(\mathbf{k}_1 - \mathbf{k}_2)$ represent, respectively, the direct and the exchange ph matrix elements of the Bonn potential in the appropriate spin-isospin channel. These formulas can be generalized to encompass the ground-state correlations as well simply by inserting into them the full free propagator G^0 and Π^0 (*i. e.,* by including both retarded and advanced components). Accordingly the polarization propagator will read

$$\Pi^{\mathrm{RPA}}(\mathbf{q}, \omega) \cong \frac{\Pi^0(\mathbf{q}, \omega)}{1 - V(\mathbf{q})\Pi^0(\mathbf{q}, \omega) - [\Pi^{(1)}(\mathbf{q}, \omega)/\Pi^0(\mathbf{q}, \omega)]}, \tag{51}$$

with

$$\Pi^{(1)}(\mathbf{q}, \omega) = -\int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} G^0(k_1 + Q) G^0(k_1) G^0(k_2 + Q) G^0(k_2) V(\mathbf{k}_1 - \mathbf{k}_2). \tag{52}$$

The HF correlations should be included, in principle, using in eq. (51) Π^{HF} in place of Π^0 and in eq. (52) G^{HF} in place of G^0 (and analogously for the TD expressions). To simplify the calculations, we shall make use of the prescription discussed in sect. 2, which amounts to replacing the bare nucleon mass with the dressed one and to shifting the energy (see the discussion above eq. (33)).

In the following we discuss the results that formulas (49) and (51) yield with the Bonn potential. We split our analysis into two parts: in the first one (subsection 4.1) we deal with the charge response in the TD approximation both in the free ($R_L^{\mathrm{TD,free}}$) and in the HF ($R_L^{\mathrm{TD,HF}}$) basis; in the second one (subsection 4.2) we explore the charge response in the full RPA, again in the free ($R_L^{\mathrm{RPA,free}}$) and in the HF ($R_L^{\mathrm{RPA,HF}}$) basis.

4.1   Tamm-Dancoff correlations

Note that in obtaining $S_{\mathrm{TD,free}}^L$ and $S_{\mathrm{TD,HF}}^L$ we have not used the normalizing factor (36), but rather the "relativized" version of it [3], namely

$$X'_{L,\mathrm{app}} = \mathcal{N} \frac{f_L^2}{\kappa \eta_F} \frac{1}{\partial \psi / \partial \lambda}, \tag{53}$$

since the TD charge responses in the free and HF bases are only approximately, although quite accurately, relativistic in the sense previously discussed in sect. 2. In (53) ψ is meant to be given by (25) and f_L^2 by (26) for the proton ($\mathcal{N} = Z$) and for the neutron ($\mathcal{N} = N$), respectively.

The *reduced* longitudinal responses $S_{\mathrm{TD,free}}^L$ and $S_{\mathrm{TD,HF}}^L$, like S_{HF}^L, should yield the same sum rule as the non-interacting RFG [16]. In Fig. 6a this is seen indeed to occur with a good accuracy. In connection with the mean energy $\bar{\lambda}$ and the variance $\bar{\sigma}$, it is of interest to recognize in Figs. 6b and c that the TD correlations alone ($R_L^{\mathrm{TD,free}}$) *do not* modify either of these two observables with respect to the predictions of the non-interacting Fermi gas, although in principle they do not have to fulfill the energy-weighted and energy-squared weighted sum rules. This outcome is reasonably clear for the variance as seen in Figs. 6–6, where R_L is displayed in HF, TD and HF-TD approximations at $q = 300, 500, 800$ and 1000 MeV/c for $k_F = 225$ MeV/c, whereas it is somewhat less evident that there is a near identity of the mean energy of the RFG and of the free basis TD responses at low momenta.

To provide an orientation for understanding both the moments of the charge response and R_L itself over a wide range of momentum transfers we start by discussing the momentum dependence of the ring and exchange diagrams. In the former case such a dependence is mainly dictated by the direct ph matrix element of the Bonn potential, whose sole dependence is upon the momentum transfer q (and this is substantial only in the isoscalar channel — in the isovector one the ph matrix element of the ρ is negligible):

$$V(q) \;\; = \;\; V_0^\sigma(q) + V_0^\omega(q)$$

$$= 4 \left\{ g_\omega^2 \Gamma_\omega^2(q) \frac{1}{q^2 + m_\omega^2} - g_\sigma^2 \Gamma_\sigma^2(q) \frac{1}{q^2 + m_\sigma^2} \right.$$

$$\left. - \left(\frac{q^2}{8m_N^2} \right) \left[g_\omega^2 \Gamma_\omega^2(q) \frac{1}{q^2 + m_\omega^2} + g_\sigma^2 \Gamma_\sigma^2(q) \frac{1}{q^2 + m_\sigma^2} \right] \right\}. \qquad (54)$$

This formula most clearly shows the battle going on in the ring framework between the σ and the ω, and won by the former, as is apparent from Fig. 6 where $V(q)$ is displayed. There it is seen that for momenta up to about 500 MeV/c a substantial attraction occurs that induces a softening of $R_L^{\text{ring,free}}$. This softening is evident in panel (b) of Figs. 6 and 6. In contrast to this, for larger momenta $V(q)$ is no longer strong enough to organize much collectivity in the nuclear motion and thus fails to affect R_L significantly (see panel (b) of Fig. 6 and 6). It should be remarked that strictly speaking formula (54) is not valid because the P-dependence of V has been ignored. On the other hand this seems to be warranted, since in the direct (ring) channel $\mathbf{P} = (\mathbf{k} - \mathbf{k}')/2$, $\mathbf{k}$ and $\mathbf{k}'$ being the hole momenta of two subsequent rings. Thus P is quite small, being bound by k_F.

Turning to the exchange we first observe that in this channel $\mathbf{P} = \mathbf{q}/2$ and therefore the corresponding terms in the Bonn potential cannot now be neglected. Moreover, the interaction lines in the exchange diagram do not depend upon q. Nevertheless, for short-range interactions, the direct and the exchange matrix elements are actually rather similar: thus the effect of the exchange and likewise the ring diagrams is no longer perceived at large q, the reason however stemming not only from the interplay among the different mesons, but also from the competition between the isoscalar ($\tau = 0$) and the isovector ($\tau = 1$) contributions, as will be illustrated in the following.

In addition to the momentum dependence it is also of importance to recall the role played by the mass of the quanta exchanged by the nucleons. Indeed, the larger the mass, the shorter the range of the force and the slower the decrease with momentum of the interaction.

On the basis of the above considerations the following comments are appropriate concerning the results displayed in the Figs. 6–6:

i) As already anticipated in the Introduction the forward-going ring diagrams, contributed to only by the σ and ω mesons, dramatically affect the charge response, however in "opposite" directions. Indeed, they lead to the occurrence of isoscalar collective modes above (the ω) and below (the σ) the QEP energy range, thus strongly depleting the charge response itself in the QEP domain. Actually, the σ taken alone would truly induce a phase transition in the RFG. However, when the σ and the ω are considered together in the ring framework, their effect is substantial but not dramatic, yielding for $q \lesssim 500$ MeV/c an $R_L^{\text{TD,free}}$ that is quenched, flattened and softened with respect to R_L^{RFG}. Then these effects fade away and at roughly 800 MeV/c little is left of them (see the panel (b) in Figs. 6–6). This prevailing of the σ over the ω in the forward-going ring diagrams is in accord with the above discussion

and clearly goes in parallel with the analogous effect occurring in the binding energy of nuclear matter.

ii) *All* of the mesons contribute to the exchange diagrams. Here the σ and ω counteract the attraction and repulsion, respectively, that they induce through the rings, thus providing an exchange-repulsion in the case of the σ and an exchange-attraction in the case of the ω (in the exchange an additional minus sign occurs). Furthermore, an important contribution that is substantial up to about 500 MeV/c, particularly in the isoscalar channel, is provided by the pion.

Where the effects on the charge response are concerned, it should be observed that at $q = 300$ MeV/c an exchange repulsion prevails. It leads to the occurrence of a *dramatic peak* in $R_L^{\text{exch,free}}$ (see Fig. 6c) already referred to in the Introduction. Fig. 6 is instrumental in grasping the origin of the peak. Specifically, in Fig. 6a the total exchange response $R_L^{\text{exch,free}}$ is displayed and compared with the one obtained using a force mediated only by the σ and ω (or the σ, ω and ρ). The pion's role appears to be truly stunning. Without it no peak would show up in the exchange response. Is this effect associated with the direct pionic contribution to $R_L^{\text{TD,free}}$ or with the interference that the pion is experiencing with the other mesons?

The answer to this question can be read in Fig. 6b where the $R_L^{\text{TD,free}}$ one would get with all of the contributions, except for the one arising from the interference of the pion with the other mesons, is compared with the full TD charge response in a free basis. The large difference between the two curves is then to be ascribed entirely to the existence in the exchange diagrams of the interferences between the pion and the other mesons. Note however that in the same figure the contribution ΔR_π due to the pion alone is also displayed and seen to be quite out of phase with that of the interference. Accordingly, the net pionic contribution to $R_L^{\text{TD,free}}$ is no longer as substantial as in the case of the pure exchange response. Yet it is sufficient, in spite of the presence of the large and attractive ring term, to bring about the hardening of the TD response seen in Fig. 6d.

For $q = 500$ MeV/c the peak is essentially gone, since the influence of the π and the σ on the one hand and of the ω on the other more or less balance each other, although some hardening of the response still takes place. It is worth observing that, in contrast to the nearly exact cancellation of their Fock contribution to the mean field, the σ and ω *do* affect the charge response via the exchange term and that this arises mainly from the σ, which acts in the same direction as the pion in shifting the maximum of R_L to larger energies for momenta up to about $q = 500$ MeV/c. For larger q the exchange becomes insignificant. This fading away is due not only to the compensation of the contributions of the different mesons (but for the ρ, which is very small by itself at large momenta), but also through an isoscalar-isovector compensation as illustrated in Fig. 6 for $q = 1$ GeV/c. In particular, there one sees that an appreciable pionic isoscalar exchange contribution still occurs in spite of the large momentum.

An additional comment may be made about the roles of the π and ρ in the

exchange diagram. We first recall that the tensor force cannot act in the charge channel [2]. Accordingly, the π and ρ will be felt in R_L only through the central part of the interaction they carry. For the π in particular, it is the short-range component (the δ-component for pointlike nucleons) which mainly comes into play, yielding, as we have already seen, a crucial repulsion at moderate momenta (indeed the pion is strongly attractive in the direct spin-longitudinal channel). In the case of the ρ, the finite-range component of the central interaction is stronger than for the pion: since it has an opposite sign to the zero-range component, a net, but much weaker, attraction mediated by the ρ thus results in the exchange channel.

In summary for the exchange contributions, we find the following: the pattern of the total TD exchange contribution to R_L is quite complex — it turns out to be repulsive through the decisive action of the π that develops mainly in the isoscalar channel, which tilts the competition between the ω and the σ in favour of the latter for q less than about 500 MeV/c. Because of this repulsion an "exchange collective mode" appears as a marked peak on the high-energy side of the response for $q = 300$ MeV/c. For larger q, the peak no longer appears and the exchange contribution becomes very small.

iii) We now come back to the delicate balance taking place in the TD framework between the ring and exchange contributions. Indeed, when evaluated in a free basis, the trend of these is to cancel one another. The cancellation however becomes complete only around $q = 800$–1000 MeV/c. On the other hand for lower momenta, as we have seen (see panel (d) of Fig. 6), a *hardening* of the charge response occurs, signaling the dominance of the exchange in the TD correlations that arises crucially from the π and σ (since this is counteracting the ω contribution).

iv) We have previously mentioned the strength of the HF mean field arising from the Bonn potential. A question to be asked is then: what happens to the TD correlations when framed in the HF basis?

Here, as before, the answer should distinguish between the range of momenta below and above 500 MeV/c. In the former instance (panel (d) of Figs. 6 and 6) a hardening, quenching and some flattening of R_L is clearly apparent. As in the case of the free basis it arises from the exchange diagrams via the π and σ. For q above about 500 MeV/c an almost pure HF response shows up instead, which arises because of a substantial weakening of the force due to a compensation taking place among both the various mesons and TD diagrams.

4.2 Ground-state correlations

We treat the gsc in this separate section because of their subtle intertwining with the forward-going amplitudes, which significantly affects the interplay between ring and exchange discussed previously.

From a glance at Figs. 6–6 it is immediately apparent that the gsc strongly enhance the impact of the ring diagrams on the charge response, at the same time

damping the influence of the exchange contributions. More specifically, as already pointed out, the gsc go in the direction to suppress a hardened and enhance a softened R_L and thus to respect the energy-weighted sum rule, as they are required to do. It is of relevance that the two effects, when they exist (namely for $q \lesssim 500$ MeV/c), have a common origin. Indeed, we have seen in Fig. 6 that the direct *ph* interaction is always *negative*. On the other hand, neglecting for a moment the exchange term, it clearly follows from (51) that an enhancement of the charge response should be expected if

$$\mathrm{Re}\Pi^0(\mathbf{q}, \omega) \approx \frac{1}{V(\mathbf{q})}. \tag{55}$$

In Fig. 6 the two sides of this equation are displayed versus ω for $q = 300$ MeV/c. It is satisfying to realize that no solution of (55) occurs if only the retarded component of $\mathrm{Re}\Pi^0$ is kept, whereas a solution exists when the advanced component (gsc) is included as well. Hence the peak on the low-energy side of Fig. 6b and the absence of the peak in Fig. 6b are explained. Note that the existence or absence of a solution of eq. (55) does not depend upon the basis (either free or HF). Note also that the peak has a width, since it is embedded in the *ph* continuum.

Furthermore, in the exchange channel the interaction turns from attractive to repulsive. The equation to be considered in this case reads

$$\mathrm{Re}\left[\frac{\Pi^{(1)}(\mathbf{q}, \omega)}{\Pi^0(\mathbf{q}, \omega)}\right] = 1. \tag{56}$$

Although eq. (56) can be solved numerically with and without the advanced components, it is of significance to realize that to the extent that

$$\Pi^{(1)}(\mathbf{q}, \omega) \approx -V(\mathbf{q})[\Pi^0(\mathbf{q}, \omega)]^2 \tag{57}$$

represents an acceptable approximation (which is the case for the present qualitative discussion), then instead of (56) one can consider (55), *except* that now the interaction is *positive*. A glance at Fig. 6 is then sufficient for concluding that gsc can prevent the occurrence of collective effects in the charge response for a *repulsive* interaction. Indeed, this is seen to be the case by comparing panels (c) of Figs. 6 and 6. Moreover, by comparing Figs. 6 to 6 with the corresponding ones in the TD approximation, it is also apparent that the gsc are no longer really operative at large momenta. Thus, as in the TD case, in RPA we witness a vanishing of the impact of the correlations for increasing momenta and for the same reasons, namely, the fight between the σ and ω in the ring channel, the fight of the π and σ against the ω, plus the isoscalar-isovector cancellation in the exchange channel.

The vanishing of the gsc at large q is clearly seen as well in panel (a) of Fig. 6 and indeed the Coulomb sum rule appears to be satisfied almost perfectly at $q = 1$ GeV/c. At lower values of q one observes in the same figure that the gsc violate the

Coulomb sum rule, but more markedly so when they are evaluated in a HF basis rather than in the free one. On the other hand, from panels (b) and (c) of Fig. 6, the mean energy and the variance of the response are seen to be essentially the same as those obtained in the TD framework.

We should now comment on the critical role played by the pion in the RPA response. Notably, in RPA a situation arises that is symmetrical, but inverse to the one occurring in the TD. Indeed, the peak that shows up in Fig. 6b for the ring approximation (now containing components going both forward and backward in time) would still be present in both $R_L^{\text{RPA,free}}$ and $R_L^{\text{RPA,HF}}$, except for the role played by the pion. This is clearly seen to happen in Fig. 6 where, as in Fig. 6b but now in the RPA, the response one would get with all of the contributions added to the free one, except for the interference of the pion itself with all of the other mesons, is displayed and compared to the full response at $q = 300$ MeV/c. A broad peak at low energy sustained by the attraction provided by the σ via the rings is still clearly present. In the same figure the π–$\sigma\omega\rho$ interference contribution is also shown together with the direct pionic contribution ΔR_π.

In contrast with the TD case, one sees that although these two terms are still out of phase, the dominance of the interference contribution is evident. One can thus conclude that at moderate momenta the pion is truly important in shaping the RPA charge response of the RFG and that it does so by interfering with the other mesons. It is accordingly true that the σ and ω, which are more strongly coupled to the nucleon than the pion, rather than washing out the impact of the latter on the charge response, are actually effective in enhancing it.

5   Invariance of the variance

The variance of the longitudinal response, which is related to the width at half-maximum of the response as shown by the RFG result (47), is defined in terms of the first and second moments of R_L. As already stressed it represents one of the basic attributes of the charge response and, in fact, any theory aiming to deal satisfactorily with the latter might be expected to account for this observable before attempting to reproduce finer details of R_L.

Actually when experimental data are analyzed in terms of the RFG model, the Fermi momentum k_F is usually fixed in such a way as to reproduce the experimental width. Following this train of thought one could therefore search for a value of k_F in the HF-RPA framework to give the same variance as that yielded by the RFG with $k_F = 225$ MeV/c, the value that produces reasonable agreement for the width of R_L when compared with experiment for ^{12}C at $q \lesssim 500$ MeV/c. Note that k_F in the interacting model is expected to be different from the value obtained using the free RFG. We have performed such a search obtaining in the HF-RPA approach the values $k_F = 196, 201, 206$ and 208 MeV/c at $q = 300, 500, 800$ and 1000 MeV/c.

In so doing we have arrived at a model of the longitudinal response that for large momenta produces the same zeroth moment (the sum rule, see Fig. 6a) and, by construction, the same second moment or variance as the RFG at any q.

Thus we see that the essential requirement of this procedure where the variance is maintained is to have a lower value for k_F in the interacting problem than one has in the free RFG model. Correspondingly, one has a lower density in the former than in the latter. Over the range of momentum transfers explored — the range where the models may be expected to be most applicable — our results show that the most important feature is a renormalization downwards of k_F by roughly 10% from the free RFG value. The residual q-dependence over the range $300 < q < 1000$ MeV/c is quite mild. Indeed we have no reason to believe that it should be present at all, as its purpose here has been simply to enforce the principle of the invariance of the variance exactly. When in future work we attempt to make detailed comparisons with experiment (*i.e.,* after the missing ingredients discussed in the Introduction have been incorporated in our model) the starting point for application of the present interacting model to ^{12}C will likely be with k_F fixed to around 200 MeV/c. An immediate consequence of this statement is that the results presented here using $k_F = 225$ MeV/c and the HF-RPA model will constitute our starting point when discussing medium-heavy nuclei, since the corresponding free RFG values of k_F are in the range 267–252 MeV/c for $q = 500$–1000 MeV/c.

Returning to Fig. 6, we see that the basic global element of distinction between the HF-RPA and RFG models accordingly rests on the first moment of the response (*i. e.,* the mean energy or energy-weighted sum rule). In Fig. 6b $\bar{\lambda}^L$ is indeed seen to be larger in HF-RPA than in the RFG almost uniformly in q (although, by comparing with Fig. 6b, the gsc are seen to quench somewhat the difference between the correlated and free mean energies).

To expand further on this issue we display in Fig. 6 the q-behaviour of the shift $\bar{\epsilon}^L$ of the maximum of the reduced charge response $S^L(\kappa, \lambda)$ with respect to that found for the RFG, $|Q^2|/2m_N$. Indeed it appears that the HF-RPA shifts are somewhat lower than those of the HF-TD model, the difference disappearing at $q \approx 1$ GeV/c, in agreement with the findings of the previous section where the gsc contribution was shown to become insignificant somewhere in between 800 and 1000 MeV/c. In the same figure we also display the HF-RPA and the HF-TD predictions for $\bar{\epsilon}^L$ for fixed $k_F = 225$ MeV/c. The difference between the results in the two schemes clearly displays the effect related to the renormalization of k_F — in fact the shift of the charge response is seen to be reduced by at least 35%, the larger reduction occurring at the lower momenta.

The general trend seen here is an increase of $\bar{\epsilon}^L$ with q up to around 800 MeV/c after which it begins to fall. The fact that $\bar{\epsilon}^L > 0$ is borne out by most experimental measurements, although the trend with q is hard to be sure of, given the current state of the data — in fact little shift is seen at all in some cases [11], while others [10] show a q-behaviour that is quite compatible with the results we have obtained

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at this (early) stage in our model-building. One thing is clear, however: the size of the shift seen in Fig. 6 is larger than that seen in the experimental studies to date. Clearly the ingredients that are still missing from our analysis should be incorporated before serious comparisons with experiemnt are made and clearly also, given the rather confused situation that still obtains on the experimental side, new measurements are essential.

The HF-RPA charge response is shown in Fig. 6 over the usual range of momenta both for k_F fixed at 225 MeV/c (corresponding to medium-heavy nuclei) and for k_F varying with q according to the principle of the invariance of the variance (and roughly representing ^{12}C, see above). The overall *quenching* of the interaction's effects on the longitudinal charge response is clear, as is its evolution with q. Note that the quenching of the contribution to the longitudinal response of the individual mesons is q-dependent as well, an outcome possibly related to the different ranges of the interaction carried by the different quanta. As an example to illustrate this point, in Fig. 6 R_L is displayed for $q = 300$ MeV/c along with the individual contributions of the π and σ. Here the renormalization of k_F is the largest and accordingly the suppression of the interaction the strongest. One sees that the pionic contribution to R_L is significantly less affected by the change in k_F than the one arising from the σ.

6 Conclusions

This paper represents a further investigation (among many) of the nuclear charge response function. It therefore seems appropriate to comment briefly on its motivations. In addition, while most of our basic conclusions were anticipated in the Introduction, there are nevertheless a few that bear amplification.

Clearly one of our motivations in the present work has been to explore R_L over a wider span of momenta than is usually done, having in mind possible future experiments to be carried out for example at M.I.T./Bates, Mainz and CEBAF. In order to accomplish this task fulfillment of Lorentz covariance is of major importance and indeed much attention has been paid by us in this connection. Furthermore, in order to provide reliable predictions for R_L at momenta up to as large as 1 GeV/c, a reliable force should be utilized. We have chosen the Bonn potential to be representative of modern approaches in this range of energies. Of course in calculating the longitudinal response the harmony between currents and forces must be preserved. Thus the predictions on R_L offered in this paper should first of all be supplemented with MEC contributions before detailed comparisons with the available experimental data are attempted. This is a task we are currently undertaking.

We have seen that the application of the principle of the invariance of the variance has led us to calculate the longitudinal response at values of k_F somewhat lower than

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those usually considered, the latter having been determined within the context of the free RFG model which should be expected to yield different results than our HF-RPA model. The consequences of this renormalization to lower values of k_F and hence to lower densities for those contributions to R_L that we have neglected in the present analysis, namely the $2p - 2h$ excitations, the MEC effects and effects from ladder diagrams, remain interesting issues to be explored further. All of these scale more rapidly by extra powers of k_F than the HF, TD and RPA contributions considered by us in the present paper. Accordingly, if we stick to the requirement of reproducing the variance of the experimental response within the HF–RPA scheme, their contributions to R_L are likely to be substantially suppressed. With respect to this principle it should also be observed that were our HF–RPA theory to be able to account for the mean energy (*i. e.,* the first moment of the experimental response) then it would *ipso facto* account as well for the second moment. This could already bring the HF–RPA framework into rather close touch with experimental data and, as a consequence, less room would be left for the contribution of the MEC, $2p - 2h$ and ladder diagrams. Our starting point in future work aimed at exploring these contributions will be to take $k_F \cong 200$ MeV/c for ^{12}C and 225 MeV/c for medium-heavy nuclei. It remains to be seen what the ultimate predictions will be for the moments of the reduced charge response (including moments higher than two that characterize the shape of the response) and how these do or do not agree with experiment. Such detailed comparisons will be attempted once the missing ingredients have been incorporated in our extended model.

Moreover, clearly even at this stage in our analysis we see that it is very desirable to have the experimental situation concerning the longitudinal response clarified, especially given its present status where it is characterized by conflicting data, since only then can definitive tests be made of our predictions.

Be it as it may, our analysis shows that, even in the limited HF–TD and HF–RPA schemes, the major reshaping of the charge response with respect to the predictions of the pure RFG model arises from a rather complex intertwining among different forward- and backward-going diagrams and among the different mesons carrying the NN interaction. In our view it is of significance that there appears to be good reason to believe that, of the latter, the pion ultimately plays the crucial role in yielding the shape and the norm of the charge response of the correlated RFG. As illustrated in the present paper this is accomplished through quite subtle aspects of the nuclear many-body problem. For example, it is clear that the interference occurring between the pion and the other mesons acquires a quite different magnitude according to whether it occurs only in forward-going processes (TD) or in the ones going backward in time as well (RPA). It appears that precisely these interference terms significantly enhance the role played by the pion in the charge response. This also suggests that the effect of the enhancement of the weak neutral current longitudinal response that we predicted [3] in the framework of a purely pionic model will survive even in the presence of other mesons.

A relevant question to be addressed for the longitudinal response obtained in the present paper within the context of the RPA-HF model concerns its sensitivity to the parameters that characterize the pieces of the Bonn potential that we have employed. There are nine such parameters altogether, namely five coupling constants (two for the ρ) and four cutoffs, *i.e.,* when the masses are taken as fixed which might be questionable in the case of the σ-meson.

The stability of the mean field with respect to moderate variations of the parameters appears to be well-established: we have indeed seen that the mean field arises mainly from the quadratic terms in the self-energy of the σ and ω and that these have the "same sign".

The situation concerning the RPA correlations is different in that there a clearcut statement cannot be made. In the direct channel the competition between the σ and ω mesons, both in forward- and backward-going rings, leads to a cancellation of the associated contributions which can be expected to be affected by variations of the force parameters. This, of course, is an effect analogous to the one that occurs for the ground-state energy. In the exchange channel the σ–ω competition is altered by the action of the π and ρ, especially the former. Therefore in this case the sensitivity to variation of the force parameters will stem not only from contributions that tend to have delicate cancellations, but also from the non-linearity in k_F contained in the pionic interferences with the σ and ω.

We are presently carrying out the task of assessing the extent of these sensitivities to details of the input potential model, despite the complexity of such an undertaking.

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Figure captions

Fig. 1 The two Feynman diagrams contributing in first order to the self-energy: (a) tadpole (direct), (b) oyster (exchange).

Fig. 2 Definition of the momenta entering the Bonn potential: accordingly, one has $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_1' = \mathbf{k}_2' - \mathbf{k}_2$ and $\mathbf{P} = (\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{k}_1' - \mathbf{k}_2')/4$.

Fig. 3 The effective mass stemming from the Hartree self-energy of the σ (dashed line) and of the ω (dot-dashed line) versus the Fermi momentum k_F. The solid line represents the Hartree effective mass arising from the total $\sigma + \omega$ exchange.

Fig. 4 The Fock self-energy of the σ (dashed line) and of the ω (dot-dashed line) versus the nucleon momentum for $k_F = 225$ MeV/c. Note the flatness of the total ($\sigma + \omega$) contribution.

Fig. 5 The free (dotted) and the HF charge responses at $q = 300$, 500, 800 and 1000 MeV/c. The HF response is displayed in three versions: with the true self-energy (8–13) (solid), with the parabolic self-energy (27,28) (dashed) and with the analytic method discussed in the text (dot-dashed). Note how close the HF curves are, but for the $q = 300$ MeV/c case at very low energy. The calculation is performed for $k_F = 225$ MeV/c, which should roughly correspond to ^{12}C.

Fig. 6 The sum rule Ξ^L, the mean energy $\bar{\lambda}^L$ and the variance $\bar{\sigma}^L$ of the charge response in HF (crosses), TD (squares) and TD on a HF basis (diamonds) compared to the free RFG results (dashed line). Note the expected fulfillment of Ξ^L in all instances, the hardening of the charge response induced by the HF field but *not* by the TD correlations (although, in principle, these do not have to fulfill the energy-weighted sum rule) and the near coincidence of the variance of the RFG with and without TD correlations. In contrast, the HF variance is significantly larger than the RFG one, but for the lowest momenta, although the difference between the two is slightly reduced by the TD correlations.

Fig. 7 The two Feynman diagrams contributing in first order to the RPA series: (a) ring (direct), (b) exchange.

Fig. 8 The momentum behaviour of the direct ph interaction of the Bonn potential (the small ρ meson contribution is neglected).

Fig. 9 The charge response in the HF and TD approximations at $q = 300$ MeV/c and $k_F = 225$ MeV/c. The dotted lines represent the free RFG response. Moreover, we display: in (a) R_L^{HF} (solid); in (b) $R_L^{\mathrm{ring,free}}$ (dash) and $R_L^{\mathrm{ring,HF}}$ (solid); in (c) $R_L^{\mathrm{exch,free}}$ (dash) and $R_L^{\mathrm{exch,HF}}$ (solid); in (d) $R_L^{\mathrm{TD,free}}$ (dash) and $R_L^{\mathrm{TD,HF}}$ (solid).

Fig. 10 The charge response in the HF and TD approximations at $q = 500$ MeV/c and $k_F = 225$ MeV/c. The meaning of the lines is explained in Fig. 6.

Fig. 11 The charge response in the HF and TD approximations at $q = 800$ MeV/c and $k_F = 225$ MeV/c. The meaning of the lines is explained in Fig. 6.

Fig. 12 The charge response in the HF and TD approximations at $q = 1$ GeV/c and $k_F = 225$ MeV/c. The meaning of the lines is explained in Fig. 6.

Fig. 13 The charge response in the TD approximation at $q = 300$ MeV/c and $k_F = 225$ MeV/c. In (a): the free RFG response (dotted) and $R_L^{\text{exch,free}}$ with the full interaction (solid), with $\sigma + \omega$ (dashed), with $\sigma + \omega + \rho$ (dot-dashed). In (b): $R_L^{\text{TD,free}}$ with the full interaction (solid) and with the full interaction minus the contribution arising from the interference of the pion with the other mesons (dashed); also shown is the contribution of the pion alone (dotted), the global contribution of σ, ω and ρ (long-dashed) and the interference of the former with the latter (dot-dashed).

Fig. 14 $R_L^{\text{exch,free}}$ (solid) and the free RFG response (dotted) in the isoscalar (a) and isovector (b) channels at $q = 1$ GeV/c and $k_F = 225$ MeV/c. Also shown are the exchange responses with only the π (dashed) and with $\sigma + \omega + \rho$ (dot-dashed).

Fig. 15 The charge response in the HF and RPA approximations at $q = 300$ MeV/c and $k_F = 225$ MeV/c. The dotted lines represent the free RFG response. Moreover, we display: in (a) R_L^{HF} (solid); in (b) $R_L^{\text{ring,free}}$ (dash) and $R_L^{\text{ring,HF}}$ (solid); in (c) $R_L^{\text{exch,free}}$ (dash) and $R_L^{\text{exch,HF}}$ (solid); in (d) $R_L^{\text{RPA,free}}$ (dash) and $R_L^{\text{RPA,HF}}$ (solid).

Fig. 16 The charge response in the HF and RPA approximations at $q = 500$ MeV/c and $k_F = 225$ MeV/c. The meaning of the lines is explained in Fig. 6.

Fig. 17 The charge response in the HF and RPA approximations at $q = 800$ MeV/c and $k_F = 225$ MeV/c. The meaning of the lines is explained in Fig. 6.

Fig. 18 The charge response in the HF and RPA approximations at $q = 1$ GeV/c and $k_F = 225$ MeV/c. The meaning of the lines is explained in Fig. 6.

Fig. 19 The solution of eq. (55) at $q = 300$ MeV/c; the curves correspond to $\text{Re}\Pi_{\text{TD}}^0$ (dashed), $\text{Re}\Pi_{\text{gsc}}^0$ (dotted) and $\text{Re}\Pi_{\text{RPA}}^0$ (solid).

Fig. 20 The sum rule Ξ^L, the mean energy $\bar{\lambda}^L$ and the variance $\bar{\sigma}^L$ of the charge response in HF (crosses), RPA (squares) and RPA on a HF basis (diamonds) compared to the free RFG results (dashed line).

Fig. 21 The charge response in the RPA approximations at $q = 300$ MeV/c and $k_F = 225$ MeV/c. For the meaning of the lines see panel (b) of Fig. 6.

Fig. 22 The sum rule Ξ^L and the mean energy $\bar{\lambda}^L$ of the charge response in HF-RPA (diamonds) and HF-TD (crosses) for $k_F = 196, 201, 206, 208$ MeV/c at $q = 300, 500, 800, 1000$ MeV/c respectively.

Fig. 23 The shift of the quasielastic peak of the reduced charge response in HF-RPA (solid line) and HF-TD (dashed line). The thin curves correspond to $k_F = 225$ MeV/c, the thick curves to the variable k_F.

Fig. 24 The free (dotted) and the HF-RPA (dashed) charge responses corresponding to $k_F = 225$ MeV/c are displayed as functions of the energy transfer. The solid curves represent the HF-RPA response functions where the values of k_F are 196 , 201, 206 and 208 MeV/c for $q = 300, 500, 800$ and 1000 MeV/c respectively.

Fig. 25 $R_L^{\mathrm{RPA,free}}$ at $q = 300$ MeV/c with the full interaction (solid), with only the π (dashed) and with only the σ (dot-dashed). In panel (a) $k_F = 225$ MeV/c and in panel (b) $k_F = 196$ MeV/c.